The Meaning of Interest Rates

Chapter 4

Econ 351 – Monetary Economics

University of Tennessee, Knoxville

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Preview

- This chapter studies why interest rates matter and how they are measured
- Present value and the yield to maturity as the most accurate measure of an interest rate
- Interest rates versus rates of return, and interest-rate risk
- Real versus nominal interest rates

A dollar today is worth more than a dollar in the future – **present value** is the tool that makes cash flows at different dates comparable.

Outline

Measuring Interest Rates

Interest Rates and Returns

Real and Nominal Interest Rates

Why Study Interest Rates?

Interest Rate as the Price of Money

- The price of using money rather than holding it

Why Do Interest Rates Exist?

- Time value of money (postponing consumption)
- Compensation for risk
- Allocating resources efficiently – what would happen if borrowing were free?



The 10-year U.S. Treasury yield since 1962; shaded bars are recessions.

Source: FRED, Federal Reserve Bank of St. Louis

Time Value of Money

A dollar today is worth more than a dollar in the future.

Example: Interest Rate = 10%

- After 1 year: $1(1.10)$
- After 2 years: $1(1.10)^2$
- After n years: $1(1.10)^n$

General Case

- Interest rate $= i$
- After n years: $1(1+i)^n$

Present Value

How much is $1 received in the future worth today?

Present Value Formula

$$PV = \frac{CF}{(1+i)^n}$$

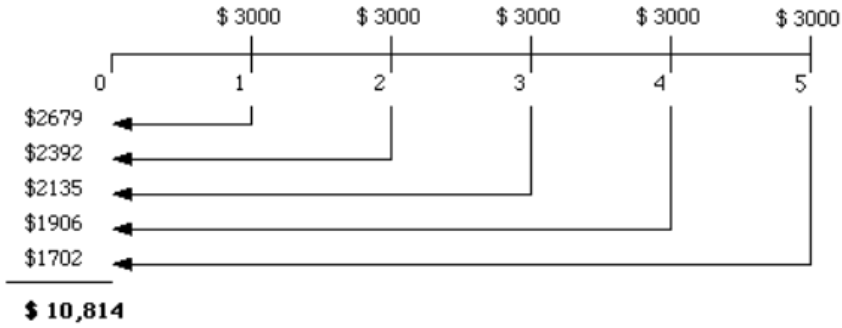
Definitions

- PV: present value
- CF: future cash flow (future value)
- i: interest rate
- n: number of years

Present Value: Timeline

Why Present Value?

- Dollar amounts at different points in time are not directly comparable
- To compare them, we convert all cash flows to the same point in time



Discounting a stream of $3,000 payments back to today.

Course figure

Example: Present Value and Economic Loss

Problem

Alice lends Bob \$10,000 today. Bob repays \$10,500 in 5 years. If the market interest rate (discount rate) is 3% per year, what is Alice's economic loss?

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Step 2: Net Present Value (NPV)

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Conclusion

In present-value terms, Alice loses about **\$943**.

Present Value of a Stream of Cash Flows

The present value of future cash flows is the current worth of a series of future cash flows, discounted to reflect the time value of money. It is widely used in bond valuation, investment analysis, and financial modeling.

Formula:

$$PV = \sum_{t=1}^{n} \frac{C_t}{(1 + r_t)^t}$$

Where:

- PV: present value of the future cash flows
- C_t: cash flow at time t (can vary for each period)
- r_t: discount rate applicable for time t (can vary)
- t: time period (e.g., year, month, or quarter)
- n: total number of periods

Example: Comparing Two Auto Loan Options

Problem

Alice wants to buy a car priced at \$50,000. The dealer requires her to finance \$30,000 over 3 years. The market interest rate is 8% per year.

Two financing options are offered:

- Option 1: pay \$1,500 interest each year and repay principal evenly over 3 years
- Option 2: pay a \$4,000 upfront fee, 0% stated interest, and repay principal evenly over 3 years

Which option is cheaper?

"0% interest" is not free money: the upfront fee is a cost too. Compare the **present value** of everything Alice pays under each option.

Comparing Present Values of the Two Options

Option 1: Present Value

Annual payment = interest + principal

$$PV_1 = \frac{1{,}500 + 10{,}000}{1.08} + \frac{1{,}500 + 10{,}000}{(1.08)^2} + \frac{1{,}500 + 10{,}000}{(1.08)^3}$$

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Option 2: Present Value

$$PV_2 = 4{,}000 + \frac{10{,}000}{1.08} + \frac{10{,}000}{(1.08)^2} + \frac{10{,}000}{(1.08)^3}$$

Comparing Present Values of the Two Options

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$$PV_2 = 4{,}000 + \frac{10{,}000}{1.08} + \frac{10{,}000}{(1.08)^2} + \frac{10{,}000}{(1.08)^3}$$

$$PV_2 \approx 29{,}770$$

Yield to Maturity (YTM)

Yield to maturity (YTM) is the discount rate that makes the present value of a bond's future cash flows equal to its current market price.

Bond Pricing Equation

$$P = \sum_{t=1}^{n} \frac{C_t}{(1 + YTM)^t}$$

- YTM is the annual return if the bond is held until maturity
- All cash flows are assumed to be reinvested at the YTM

YTM: Zero-Coupon Bond

A zero-coupon bond has a face value of \$1,000 and is currently priced at \$800. The bond matures in 5 years. What is the yield to maturity (YTM)?

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Answer

$$P = \frac{F}{(1 + YTM)^n}$$

$$YTM = \left(\frac{F}{P}\right)^{\frac{1}{n}} - 1 = \left(\frac{1000}{800}\right)^{\frac{1}{5}} - 1 \approx 4.56\%$$

With a single payment, the YTM can be solved in closed form: it is the constant annual rate that grows \$800 into \$1,000 in 5 years.

YTM: Premium Bond

A bond has a face value of \$1,000, a coupon rate of 8% (paid annually), and a market price of \$1,100. The bond matures in 5 years. Calculate the yield to maturity (YTM).

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Answer

$$C = 1000 \times 0.08 = 80$$

$$P = \sum_{t=1}^{5} \frac{80}{(1 + YTM)^t} + \frac{1000}{(1 + YTM)^5}$$

$$YTM \approx 6.41\%$$

The price is **above** face value, so the YTM (6.41%) is **below** the coupon rate (8%).

YTM: Floating-Rate Bond

A floating-rate bond has a face value of \$1,000 and pays LIBOR + 1% annually. Current LIBOR is 2%, and the bond matures in 3 years. LIBOR is expected to increase by 0.5% each year. The bond price is \$1,000. Calculate the yield to maturity (YTM).

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Answer

$$P = \frac{30}{(1 + YTM)} + \frac{35}{(1 + YTM)^2} + \frac{1040}{(1 + YTM)^3}$$

$$YTM \approx 3.48\%$$

Coupons are 30, 35, and 40 (3%, 3.5%, 4% of face value) as LIBOR rises; the YTM is a single rate that summarizes all three.

YTM: Perpetual Bond

A perpetual bond pays an annual coupon of \$50 and is currently priced at \$800. Calculate the yield to maturity (YTM).

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A perpetual bond pays an annual coupon of \$50 and is currently priced at \$800. Calculate the yield to maturity (YTM).

Answer

$$YTM = \frac{C}{P} = \frac{50}{800} = 0.0625 = 6.25\%$$

For a perpetuity, $P = C/i$: price and yield move in **opposite** directions – a result that holds for all bonds.

Mortgage Burden

Scenario

- Home size: 1,700 sq ft (about 160 m^2); price: \$400 per sq ft $\Rightarrow$ home price $= 1{,}700 \times 400 = \$680{,}000$
- Down payment: 20% $\Rightarrow 0.20 \times 680{,}000 = \$136{,}000$ paid upfront
- 30-year fixed-rate mortgage, annual interest rate: 6.5%

Monthly payment

$$L = 0.80 \times 680{,}000 = \$544{,}000, \quad i = \frac{0.065}{12}, \quad n = 360$$

$$544{,}000 = \sum_{t=1}^{360} \frac{CF}{(1+i)^t} \quad \Longrightarrow \quad CF = L \cdot \frac{i}{1-(1+i)^{-n}} \approx \$3{,}440 \text{ per month}$$

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Rate of Return

The rate of return equals the payments to the owner plus the change in value, expressed as a fraction of the purchase price:

$$RET = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

Components

- $\frac{C}{P_t} = i_c$: current yield
- $\frac{P_{t+1} - P_t}{P_t} = g$: rate of capital gain

The return on a bond is **not** the same as its interest rate: it also depends on what happens to the bond's price while you hold it.

YTM and Bond Price

- Bond prices and yields move in opposite directions
- Consider a bond with:
  - Face value: \$1,000
  - Coupon rate: 10%
  - Maturity: 10 years
- When the coupon rate equals the yield to maturity, the bond price equals its face value

$$P = \sum_{t=1}^{10} \frac{1000 \times 0.10}{(1+i)^t} + \frac{1000}{(1+i)^{10}}$$

Price of bond (\$)	YTM (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

Interest Rates, Returns, and Interest-Rate Risk

Interest Rates vs. Returns

- The return equals the YTM only when the bond is held to maturity
- When interest rates rise, bond prices fall, causing capital losses for investors who sell before maturity
- Bonds with longer maturities experience larger price changes for a given change in interest rates

Maturity and Interest-Rate Risk

- Long-term bonds have more volatile prices and returns than short-term bonds
- A bond has no interest-rate risk when its maturity equals the holding period

Even a default-free Treasury bond is risky if you may have to sell it before maturity – this is **interest-rate risk**, and it grows with maturity.

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Real and Nominal Interest Rates

Real and Nominal Interest Rates

Definitions

- **Nominal interest rate**: makes no allowance for inflation
- **Real interest rate**: adjusted for changes in the price level and therefore more accurately reflects the cost of borrowing

Types of Real Interest Rates

- **Ex ante real interest rate**: adjusted for expected changes in the price level
- **Ex post real interest rate**: adjusted for actual changes in the price level

Borrowers and lenders care about the **real** rate: a 50% nominal rate is cheap if prices are rising 60% a year.

Fisher Equation

Fisher Equation

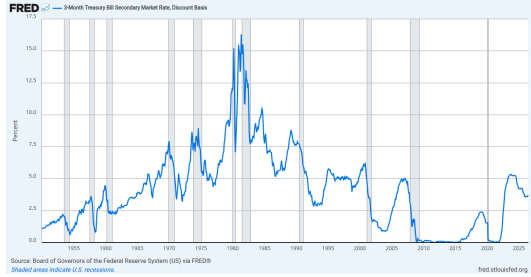
$$i = i_r + \pi^e$$

Exact Fisher Equation

$$1 + i = (1 + i_r)(1 + \pi^e)$$

$$i = i_r + \pi^e + i_r\pi^e$$

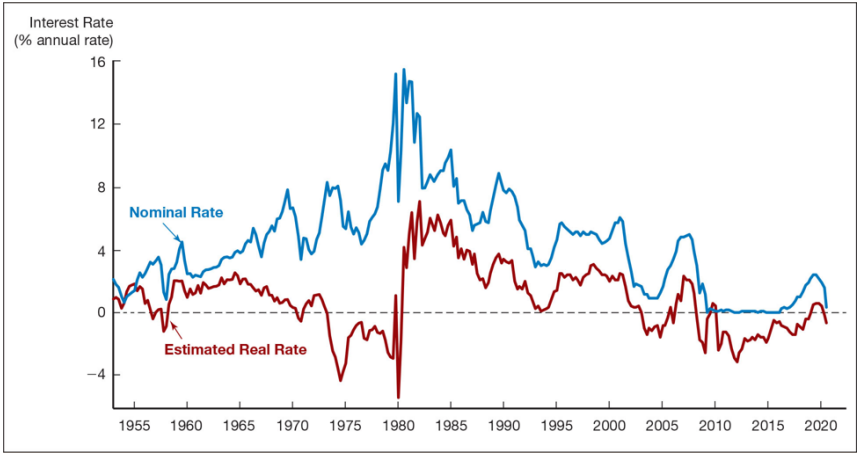
- i: nominal interest rate
- i_r: real interest rate
- π^e: expected inflation rate



The nominal 3-month Treasury bill rate since 1950 – high in the inflationary 1970s–80s, near zero after 2008.

Source: FRED, Federal Reserve Bank of St. Louis

Real and Nominal Interest Rates in the U.S.



Nominal and estimated (ex post) real three-month Treasury bill rates, 1953–2020.

Source: Mishkin, The Economics of Money, Banking, and Financial Markets

Chapter Summary

- Present value converts future cash flows into today's dollars: $PV = CF/(1+i)^n$
- The yield to maturity is the discount rate that equates a bond's price with the present value of its cash flows; price and yield move in opposite directions
- The rate of return = current yield + rate of capital gain; it equals the YTM only if the bond is held to maturity
- Longer-maturity bonds carry more interest-rate risk
- The real interest rate adjusts the nominal rate for expected inflation (Fisher equation: $i = i_r + \pi^e$)