Capital Accumulation as a Source of Growth

Chapter 9

Econ 313 – Intermediate Macroeconomics

University of Tennessee, Knoxville

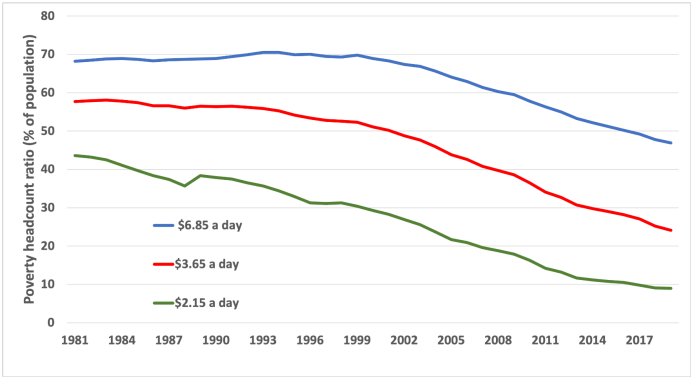
July 21, 2026

In This Chapter, You Will Learn:

- About the closed-economy Solow model
- How a country's standard of living depends on its saving rate
- How to use the "Golden Rule" to find the optimal saving rate and capital stock

Why growth matters (1 of 2)

Economic growth raises living standards and reduces poverty...



Source: World Bank, Poverty and Inequality Platform

Why growth matters (2 of 2)

Anything that affects the long-run rate of economic growth—even by a tiny amount—will have huge effects on living standards in the long run.

Annual growth rate	After 25 years	After 50 years	After 100 years
2.0%	64.0%	169.2%	624.5%
2.5%	85.4%	243.7%	1,081.4%

The lessons of growth theory

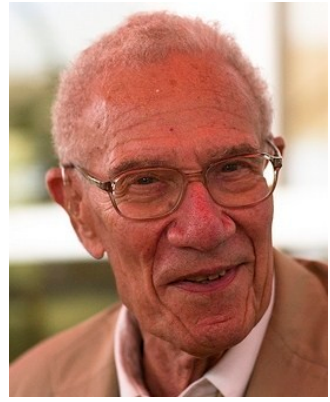
...can make a positive difference in the lives of hundreds of millions of people.

These lessons help us

- understand why poor countries are poor
- design policies that can help them grow
- learn how our own growth rate is affected by shocks and our government's policies

The Solow model

- It is named after Robert Solow, who won the Nobel Prize for contributions to the study of economic growth.
- It is
  - widely used in policymaking
  - a benchmark against which most recent growth theories are compared
- It looks at the determinants of economic growth and the standard of living in the long run.



Robert M. Solow
(1924–2023)
Nobel laureate, 1987.
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How the Solow model is different from Chapter 3's model, part 1

1. K is no longer fixed: Investment causes it to grow, depreciation causes it to shrink.
2. L is no longer fixed: population growth (Chapter 9)
3. The consumption function is simpler.

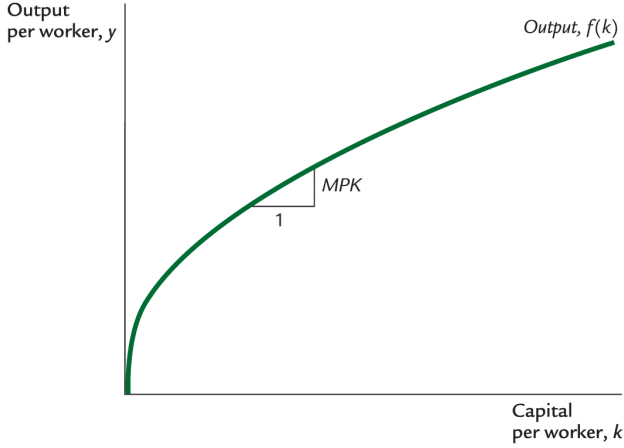
How the Solow model is different from Chapter 3's model, part 2

4. No G or T (only to simplify presentation; we can still do fiscal policy experiments)
5. Cosmetic differences

The production function (1 of 2)

- In aggregate terms: $Y = F(K, L)$
- Define: $y = Y/L$ = output per worker
  $k = K/L$ = capital per worker
- Assume constant returns to scale:
  $zY = F(zK, zL)$ for any $z > 0$
- Pick $z = 1/L$. Then
  - $Y/L = F(K/L, 1)$
  - $y = F(k, 1)$
  - $y = f(k)$, where $f(k) = F(k, 1)$

The production function (2 of 2)



Note: This production function exhibits diminishing *MPK*.

The national income identity

- $Y = C + I$ (remember, no G)
- In "per worker" terms:

$$y = c + i,$$

where $c = C/L$ and $i = I/L$.

The consumption function

- $s =$ the saving rate, the fraction of income that is saved (s is an exogenous parameter)

Note: s is the *only* lowercase variable that is *not equal to* its uppercase version divided by L.

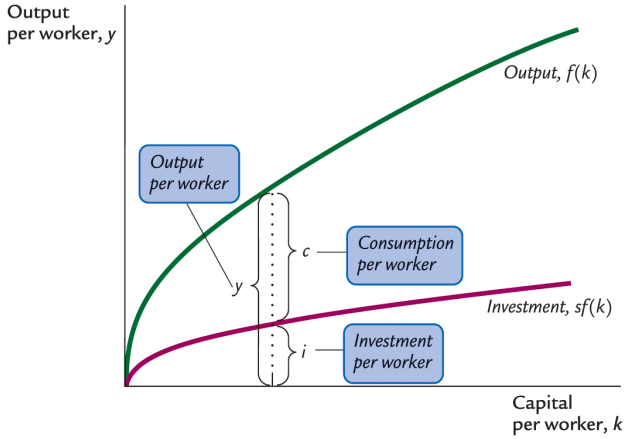
- Consumption function: $c = (1 - s)y$ *(per worker)*

Saving and investment

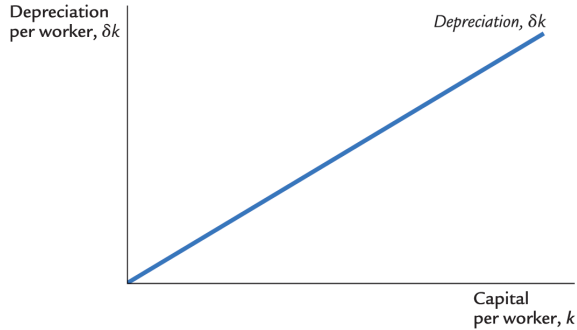
- Saving (per worker) $= y - c = y - (1 - s)y = sy$
- National income identity is $y = c + i$
  Rearrange to get $i = y - c = sy$
  (investment = saving, as in Chapter 3!)
- Using the results above,

$$i = sy = sf(k)$$

Output, consumption, and investment



Depreciation



δ = the rate of depreciation
= the fraction of the capital stock that wears out each period

Capital accumulation

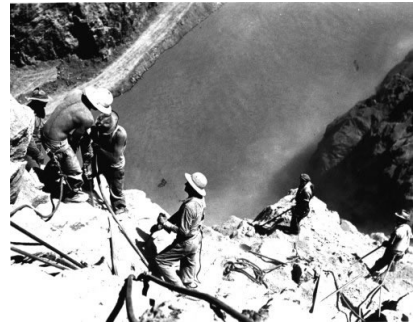
The basic idea: Investment increases the capital stock; depreciation reduces it.

change in capital stock = investment − depreciation

$$\Delta k = i - \delta k$$

Since $i = sf(k)$, this becomes:

$$\boxed{\Delta k = sf(k) - \delta k}$$



Hoover Dam under construction, c. 1934 — physical capital being accumulated.

Bureau of Reclamation / Wikimedia, PD

The equation of motion for capital (k)

$$\boxed{\Delta k = sf(k) - \delta k}$$

- The Solow model's central equation
- Determines behavior of capital over time...
- ...which, in turn, determines behavior of all the other endogenous variables because they all depend on k.
- Example:
  - income per person: $y = f(k)$
  - consumption per person: $c = (1 - s) f(k)$

The steady state, part 1

$$\boxed{\Delta k = sf(k) - \delta k}$$

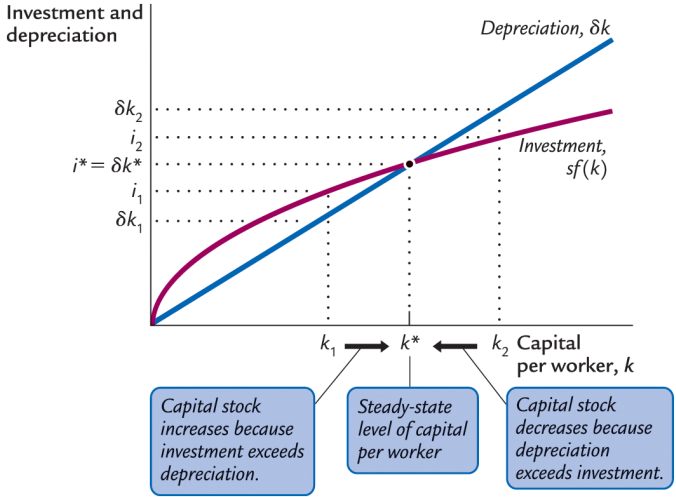
If investment is just enough to cover depreciation [$sf(k) = \delta k$], then capital per worker will remain constant:

$$\Delta k = 0.$$

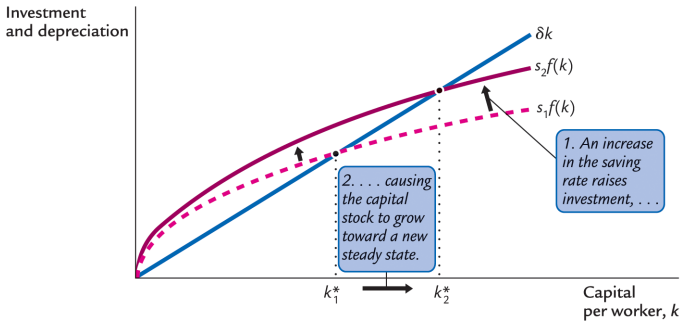
This occurs at one value of k, denoted k^*, called the **steady-state capital stock**.

▸ Solve one numerically

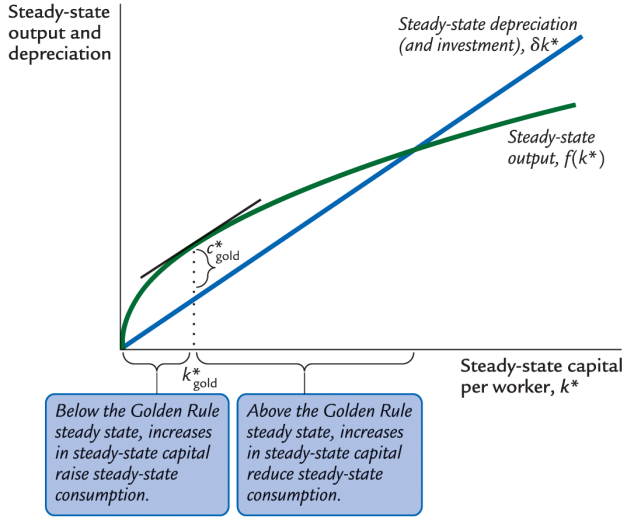
The steady state, part 2



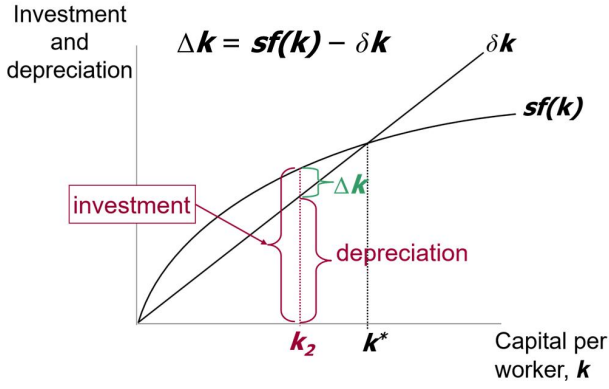
Moving toward the steady state, part 1



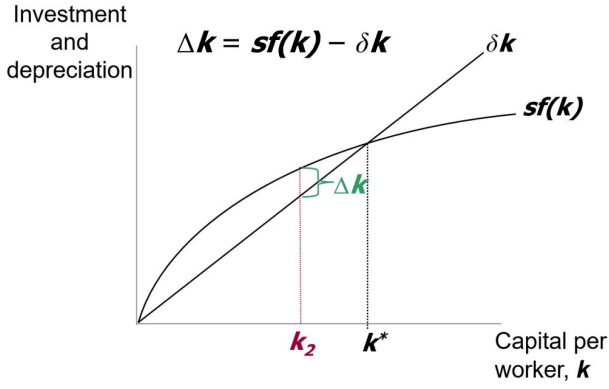
Moving toward the steady state, part 2



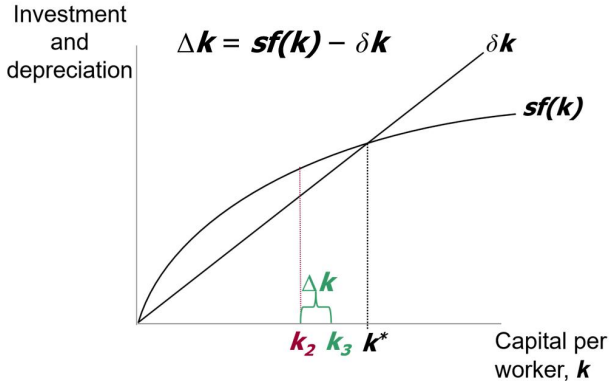
Moving toward the steady state, part 3



Moving toward the steady state, part 4

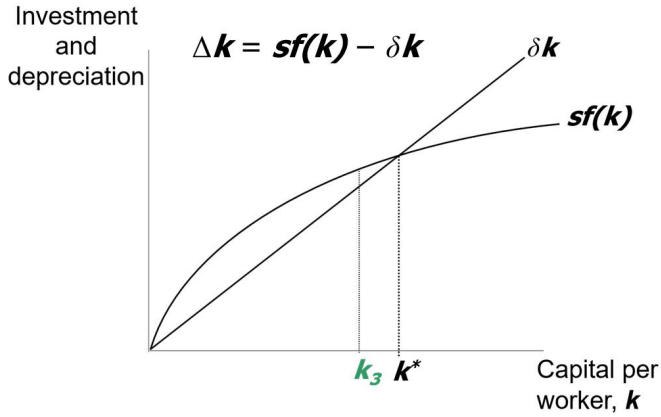


Moving toward the steady state, part 5



Moving toward the steady state, part 6

Summary:
As long as $k < k^*$, investment will exceed δ, and k will continue to grow toward k^*.



NOW YOU TRY: Approaching k^* from above

Draw the Solow model diagram, labeling the steady state k^*.

On the horizontal axis, pick a value greater than k^* for the economy's initial capital stock. Label it k_1.

Show what happens to k over time.

Does k move toward the steady state or away from it?

A numerical example, part 1

Production function (aggregate):

$$Y = F(K, L) = \sqrt{K \cdot L} = K^{1/2} L^{1/2}$$

To derive the per-worker production function, divide through by L:

$$\frac{Y}{L} = \frac{K^{1/2} L^{1/2}}{L} = \left(\frac{K}{L}\right)^{1/2}$$

Then substitute $y = Y/L$ and $k = K/L$ to get

$$y = f(k) = k^{1/2}$$

A numerical example, part 2

Assume:

- $s = 0.3$
- $\delta = 0.1$
- initial value of $k = 4.0$

Approaching the steady state: A numerical example

Assumptions: $y = \sqrt{k}$; $s = 0.3$; $\delta = 0.1$; initial $k = 4.0$

Year	k	y	c	i	δk	Δk
1	4.000	2.000	1.400	0.600	0.400	0.200
2	4.200	2.049	1.435	0.615	0.420	0.195
3	4.395	2.096	1.467	0.629	0.440	0.189
4	4.584	2.141	1.499	0.642	0.458	0.184
5	4.768	2.184	1.529	0.655	0.477	0.178
10	5.602	2.367	1.657	0.710	0.560	0.150
25	7.321	2.706	1.894	0.812	0.732	0.080
100	8.962	2.994	2.096	0.898	0.896	0.002
∞	9.000	3.000	2.100	0.900	0.900	0.000

NOW YOU TRY: Solve for the steady state

Continue to assume
$s = 0.3$, $\delta = 0.1$, and $y = k^{1/2}$

Use the equation of motion

$$\Delta k = s f(k) - \delta k$$

to solve for the steady-state values of k, y, and c.

NOW YOU TRY: Solve for the steady state, answers

$\Delta k = 0$ definition of steady state
$s f(k^*) = \delta k^*$ eq'n of motion with $\Delta k = 0$
$0.3\sqrt{k^*} = 0.1\, k^*$ using assumed values

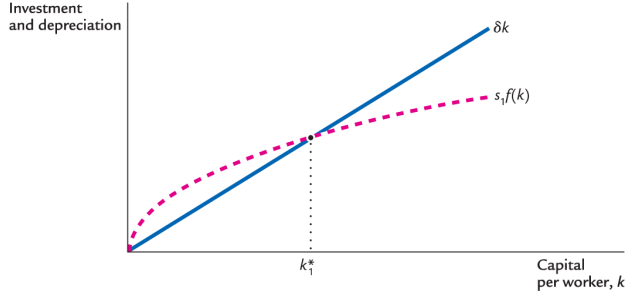
$$3 = \frac{k^*}{\sqrt{k^*}} = \sqrt{k^*}$$

Solve to get: $k^* = 9$ and $y^* = \sqrt{k^*} = 3$

Finally, $c^* = (1 - s)\, y^* = 0.7 \times 3 = 2.1$

An increase in the saving rate

An increase in the saving rate raises investment...
...causing k to grow toward a new steady state:



Prediction: Countries with higher rates of saving and investment

- The Solow model predicts that countries with higher rates of saving and investment will have higher levels of capital and income per worker in the long run.
- Are the data consistent with this prediction?

The Golden Rule: Introduction

- Different values of s lead to different steady states. How do we know which is the "best" steady state?
- The "best" steady state has the highest possible consumption per person: $c^* = (1 - s)\, f(k^*)$.
- An increase in s
  - leads to higher k^* and y^*, which raises c^*
  - reduces consumption's share of income $(1 - s)$, which lowers c^*.
- So, how do we find the s and k^* that maximize c^*?

The Golden Rule capital stock, part 1

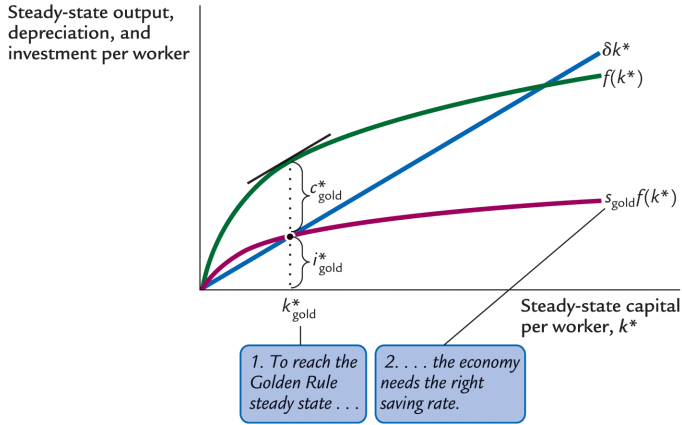
k^*_{gold} = the **Golden Rule level of capital**, the steady-state value of k that maximizes consun

To find it, first express c^* in terms of k^*:

$$
\begin{aligned}
c^* &= y^* - i^* \\
&= f(k^*) - i^* \\
&= f(k^*) - \delta k^*
\end{aligned}
\qquad
\left\{
\begin{array}{l}
\text{In the steady state:} \\
i^* = \delta k^* \\
\text{because } \Delta k = 0.
\end{array}
\right.
$$

The Golden Rule capital stock, part 2

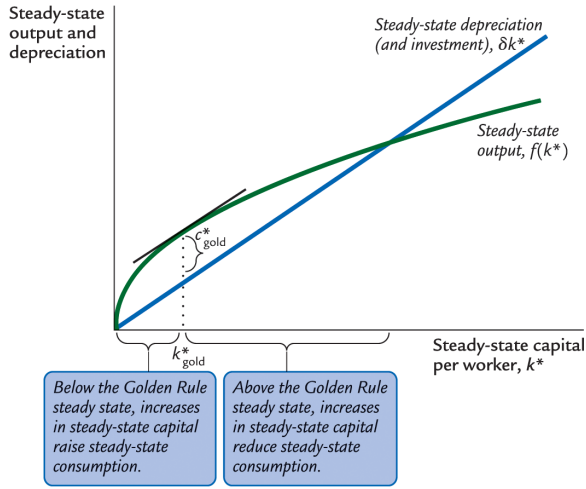
Then, graph $f(k^*)$ and δk^*, looking for the point where the gap between them is biggest.



The Golden Rule capital stock, part 3

$c^* = f(k^*) - \delta k^*$ is biggest where the slope of the production function equals the slope of the depreciation line:

$$MPK = \delta$$



The transition to the Golden Rule steady state

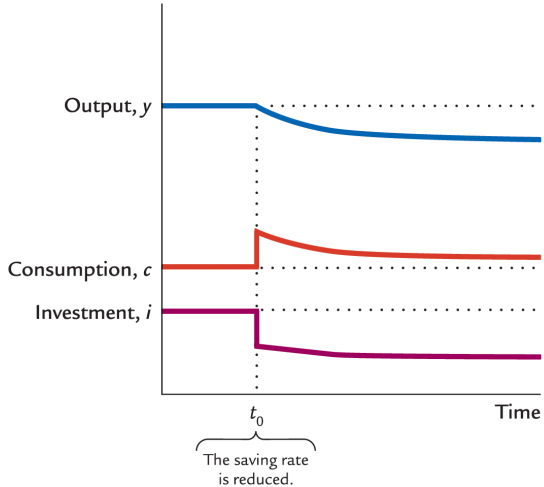
- The economy does not have a tendency to move toward the Golden Rule steady state.
- Achieving the Golden Rule requires that policymakers adjust s.
- This adjustment leads to a new steady state with higher consumption.
- But what happens to consumption during the transition to the Golden Rule?

Starting with too much capital

If $k^* > k^*_{\text{gold}}$

then increasing c^* requires a fall in s.

In the transition to the Golden Rule, consumption is higher at all points in time.

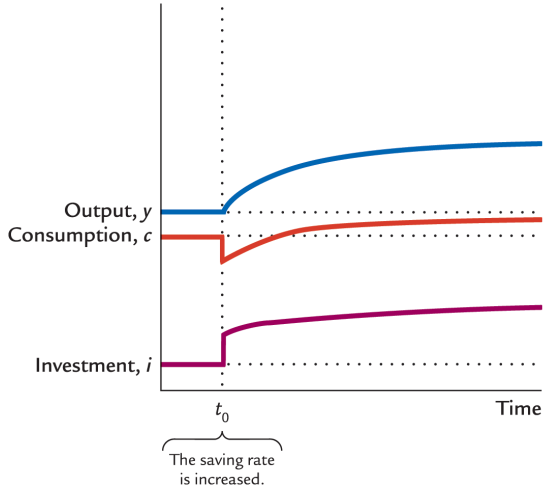


Starting with too little capital

If $k^* < k^*_{\text{gold}}$

then increasing c^* requires an increase in s.

Future generations enjoy higher consumption, but the current one experiences an initial drop in consumption.



Chapter Summary, Part 1

- The Solow growth model shows that, in the long run, a country's standard of living depends:
  - positively on its saving rate
- An increase in the saving rate leads to:
  - higher output in the long run
  - faster growth temporarily
  - but not faster steady-state growth

Chapter Summary, Part 2

- If the economy has more capital than the Golden Rule level, then reducing saving will increase consumption at all points in time, making all generations better off.
- If the economy has less capital than the Golden Rule level, then increasing saving will increase consumption for future generations, but reduce consumption for the present generation.

Appendix: Solow steady state — solved numerically

Setup. Cobb–Douglas production, per-worker form:

$$y = f(k) = k^{1/2}.$$

Parameters: saving rate $s = 0.3$, depreciation $\delta = 0.1$.
Step 1 — solve for k^*. In steady state, $s f(k^*) = \delta k^*$:

$$0.3 \cdot (k^*)^{1/2} = 0.1 \cdot k^* \implies (k^*)^{1/2} = 3 \implies \boxed{k^* = 9}.$$

Step 2 — the rest of the steady state.

$$y^* = (k^*)^{1/2} = 3, \quad c^* = (1 - s)\, y^* = 0.7 \cdot 3 = 2.1, \quad i^* = s y^* = 0.9.$$

Check: $\delta k^* = 0.1 \cdot 9 = 0.9 = i^*$.

back

Appendix — what if the saving rate rises to $s = 0.4$?

Same production function and δ, new $s = 0.4$:

$$0.4 \cdot (k^*)^{1/2} = 0.1 \cdot k^* \implies (k^*)^{1/2} = 4 \implies k^* = 16.$$

$$y^* = 4, \quad c^* = 0.6 \cdot 4 = 2.4, \quad i^* = 1.6.$$

	$s = 0.3$	$s = 0.4$	change
k^*	9	16	$+78\%$
y^*	3	4	$+33\%$
c^*	2.1	2.4	$+14\%$

Higher $s \Rightarrow$ more k^* and y^*. But c^* eventually falls if we save *too much* (Golden Rule).

back

Appendix — where is the Golden Rule?

Consumption per worker at steady state:

$$c^*(s) = (1-s)\, f(k^*(s)) = f(k^*) - \delta k^*.$$

Maximize over k^*: set $f'(k^*) = \delta$:

$$\tfrac{1}{2}(k^*_{\text{gold}})^{-1/2} = 0.1 \implies (k^*_{\text{gold}})^{1/2} = 5 \implies k^*_{\text{gold}} = 25.$$

Solving for the Golden-Rule saving rate:

$$s_{\text{gold}} \cdot 5 = 0.1 \cdot 25 \implies s_{\text{gold}} = 0.5.$$

And $c^*_{\text{gold}} = 0.5 \cdot 5 = 2.5$ (higher than either 2.1 or 2.4 above).

So $s = 0.3$ and $s = 0.4$ are both *below* Golden Rule. Raising s toward 0.5 keeps raising steady-state consumption. Going past 0.5 would "over-save" — dynamically inefficient.

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