National Income: How It Is Earned

Chapter 3

Econ 313 – Intermediate Macroeconomics

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In This Chapter, You Will Learn:

- What determines the economy's total output/income
- How the prices of the factors of production are determined
- How total income is distributed
- The growing gap between rich and poor

Outline of model

A closed economy, market-clearing model

- Supply side
  - factor markets (supply, demand, price)
  - determination of output/income
- Demand side
  - determinants of C, I, and G
- Equilibrium
  - goods market
  - loanable funds market

Factors of production

$\boldsymbol{K}$ = capital:
tools, machines, and structures used in production

$\boldsymbol{L}$ = labor:
the physical and mental efforts of workers

The production function: $Y = F(K, L)$

- Shows how much output (Y) the economy can produce from K units of capital and L units of labor
- Reflects the economy's level of technology
- Reflects how the existing technology transforms capital and labor into output
- Technological improvements result in more output from the same quantity of inputs
- Exhibits constant returns to scale

Returns to scale: A review

Initially $Y_1 = F(K_1, L_1)$

Scale all inputs by the same factor z:
$K_2 = zK_1$ and $L_2 = zL_1$
(for example: if $z = 1.2$, then all inputs are increased by 20%)

What happens to output, $Y_2 = F(K_2, L_2)$?

- If **constant returns to scale**, $Y_2 = zY_1$
- If **increasing returns to scale**, $Y_2 > zY_1$
- If **decreasing returns to scale**, $Y_2 < zY_1$

Returns to scale: Example 1

$$
\begin{aligned}
F(K, L) &= \sqrt{KL} \\
F(zK, zL) &= \sqrt{(zK)(zL)} \\
&= \sqrt{z^2 KL} \\
&= \sqrt{z^2}\sqrt{KL} \\
&= z\sqrt{KL} \\
&= zF(K, L)
\end{aligned}
$$

constant returns to scale for any $z > 0$

Returns to scale: Example 2

$$F(K, L) = K^2 + L^2$$

$$F(zK, zL) = (zK)^2 + (zL)^2$$

$$= z^2(K^2 + L^2)$$

$$= z^2 F(K, L)$$

increasing returns to scale for any $z > 1$

NOW YOU TRY: Returns to scale

Determine whether each of these production functions has constant, decreasing, or increasing returns to scale:

(a) $F(K, L) = \dfrac{K^2}{L}$

(b) $F(K, L) = K + L$

NOW YOU TRY: Answers, part (a)

$$F(K, L) = \frac{K^2}{L}$$

$$F(zK, zL) = \frac{(zK)^2}{zL} = \frac{z^2K^2}{zL} = z\frac{K^2}{L}$$

$$= zF(K, L)$$

constant returns to scale for any $z > 0$

NOW YOU TRY: Answers, part (b)

$$F(K, L) = K + L$$

$$F(zK, zL) = zK + zL$$

$$= z(K + L)$$

$$= zF(K, L)$$

constant returns to scale for any $z > 0$

Assumptions

1. Technology is fixed.
2. The economy's supplies of capital and labor are fixed at:

$$K = \bar{K} \text{ and } L = \bar{L}.$$

Determining GDP

Output is determined by the fixed factor supplies and the fixed state of technology:

$$\bar{Y} = F\left(\bar{K}, \bar{L}\right).$$

The distribution of national income

determined by **factor prices**, the prices per unit firms pay for the factors of production

- wage = price of L
- **rental rate** = price of K

Notation

$\boldsymbol{W}$ = nominal wage
$\boldsymbol{R}$ = nominal rental rate
$\boldsymbol{P}$ = price of output
$\boldsymbol{W/P}$ = real wage
(measured in units of output)
$\boldsymbol{R/P}$ = real rental rate

How factor prices are determined

- Factor prices are determined by supply and demand in factor markets.
- Recall that the supply of each factor is fixed.
- What about demand?

Demand for labor

- Assume that markets are competitive: Each firm takes W, R, and P as given.
- Basic idea: A firm hires each unit of labor if the cost does not exceed the benefit.
  - cost = real wage
  - benefit = marginal product of labor

Marginal product of labor (MPL)

Definition:
The extra output the firm can produce using an additional unit of labor (holding other inputs fixed):

$$MPL = F(K, L + 1) - F(K, L)$$

NOW YOU TRY: Compute and graph MPL

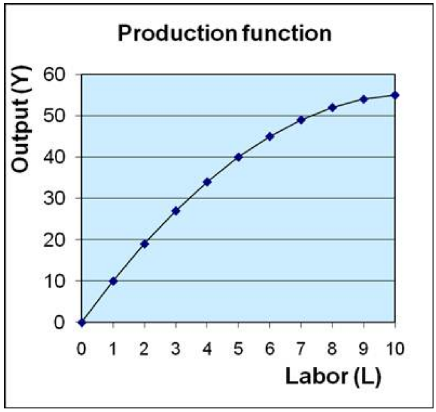
(a) Determine *MPL* at each value of *L*.

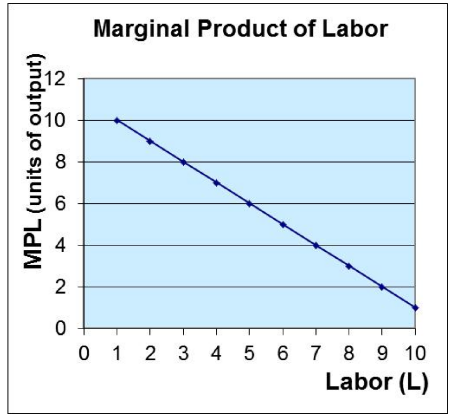
(b) Graph the production function.

(c) Graph the *MPL* curve with *MPL* on the vertical axis and *L* on the horizontal axis.

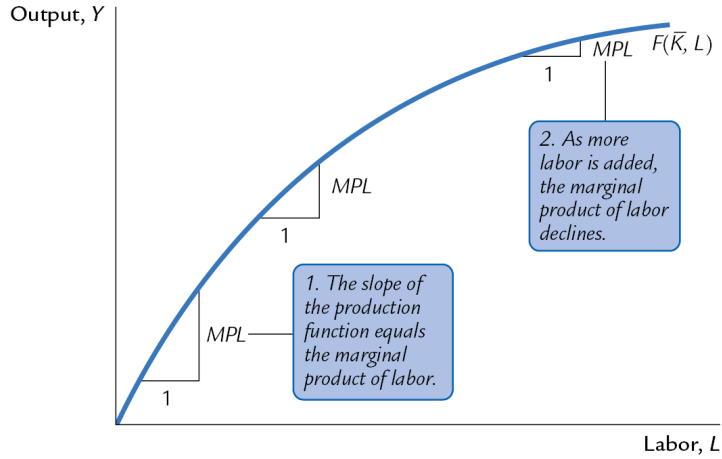
L	*Y*	*MPL*
0	0	n.a.
1	10	?
2	19	?
3	27	8
4	34	?
5	40	?
6	45	?
7	49	?
8	52	?
9	54	?
10	55	?

NOW YOU TRY: Compute and graph MPL, Answers





MPL and the production function



Diminishing marginal returns

- As one input is increased (holding other inputs constant), its marginal product falls.
- Intuition:
  If L increases while holding K fixed, machines per worker fall, worker productivity falls.

NOW YOU TRY: Identifying diminishing returns

Which of these production functions have diminishing marginal returns to labor?

(a) $F(K, L) = 2K + 15L$

(b) $F(K, L) = \sqrt{KL}$

(c) $F(K, L) = 2\sqrt{K} + 15\sqrt{L}$

NOW YOU TRY: Identifying diminishing returns, answers

Which of these production functions have diminishing marginal returns to labor?

(a) $F(K, L) = 2K + 15L$
No, $MPL = 15$ for all L

(b) $F(K, L) = \sqrt{KL}$
Yes, MPL falls as L rises

(c) $F(K, L) = 2\sqrt{K} + 15\sqrt{L}$
Yes, MPL falls as L rises

NOW YOU TRY: MPL and labor demand

Suppose $W/P = 6$.

- If $L = 3$, should the firm hire more or less labor? Why?
- If $L = 7$, should the firm hire more or less labor? Why?

L	Y	MPL
0	0	n.a.
1	10	10
2	19	9
3	27	8
4	34	7
5	40	6
6	45	5
7	49	4
8	52	3
9	54	2
10	55	1

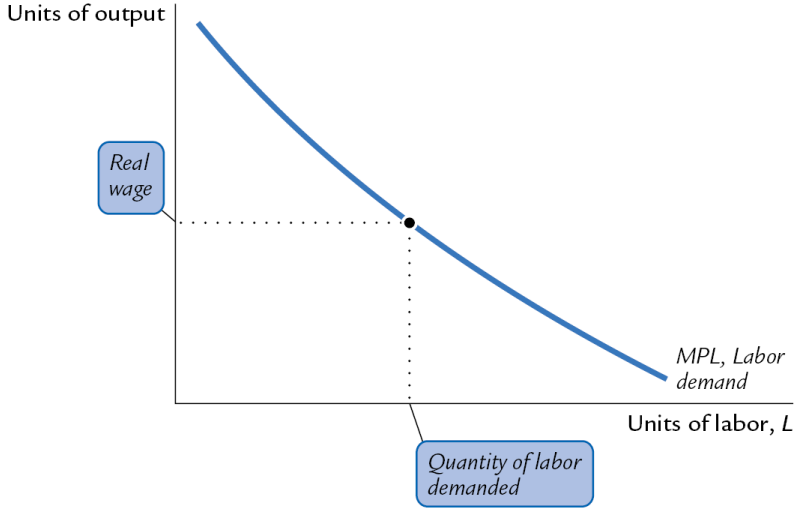
NOW YOU TRY: MPL and labor demand, answers

Suppose $W/P = 6$.

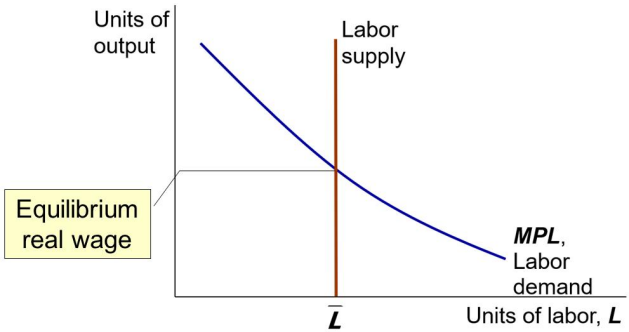
- If $L = 3$, should the firm hire more or less labor? Why?
  Answer: More because the benefit of the fourth worker ($MPL = 7$) exceeds its cost ($W/P = 6$)
- If $L = 7$, should the firm hire more or less labor? Why?
  Answer: Less because the seventh worker adds $MPL = 4$ units of output but costs the firm $W/P = 6$.

L	Y	MPL
0	0	n.a.
1	10	10
2	19	9
3	27	8
4	34	7
5	40	6
6	45	5
7	49	4
8	52	3
9	54	2
10	55	1

MPL and the demand for labor



The equilibrium real wage

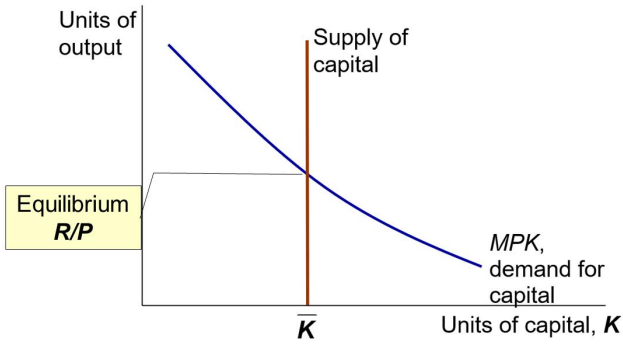


The real wage adjusts to equate labor demand with supply.

Determining the rental rate

- We have just seen that $MPL = W/P$.
- The same logic shows that $MPK = R/P$:
  - Diminishing returns to capital: MPK falls as K rises.
  - The MPK curve is the firm's demand curve for renting capital.
  - Firms maximize profits by choosing K such that $MPK = R/P$.

The equilibrium real rental rate



The real rental rate adjusts to equate demand for capital with supply.

The neoclassical theory of distribution

- States that each factor input is paid its marginal product
- A good starting point for thinking about income distribution

How income is distributed to L and K

$$\text{Total labor income} = \frac{W}{P}\bar{L} = MPL \times \bar{L}$$

$$\text{Total capital income} = \frac{R}{P}\bar{K} = MPK \times \bar{K}$$

If the production function has constant returns to scale, then

$$\bar{Y} = MPL \times \bar{L} + MPK \times \bar{K}$$

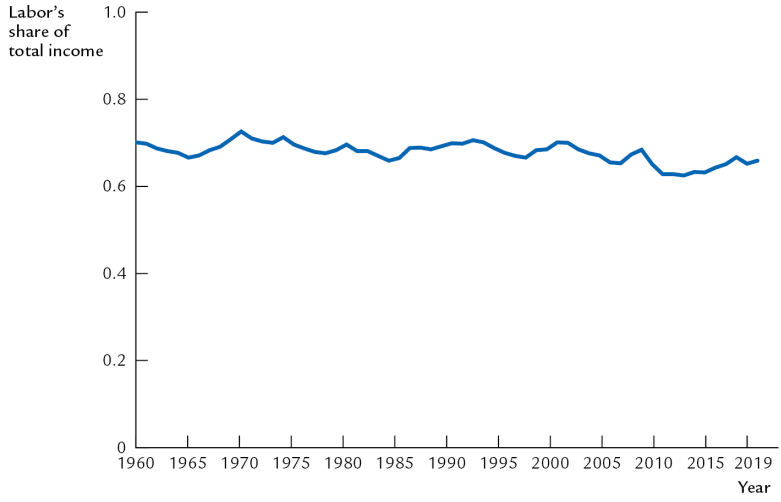
national income	labor income	capital income



Ford assembly line, 1913. Every unit of $\bar{Y}$ comes from combining $\bar{L}$ (workers) and $\bar{K}$ (the line, tools, factory).

Wikimedia, PD-US

The ratio of labor income to total income in the United States, 1960–2019



The Cobb–Douglas production function (1 of 2)

- The Cobb–Douglas production function has constant factor shares:
  - $\alpha =$ capital's share of total income:
  - capital income $= MPK \times K = \alpha Y$
  - labor income $= MPL \times L = (1 - \alpha) Y$
- The Cobb–Douglas production function is:

$$Y = AK^{\alpha} L^{1-\alpha},$$

where A represents the level of technology.

The Cobb–Douglas production function (2 of 2)

Each factor's marginal product is proportional to its average product:

$$MPK = \alpha A K^{\alpha-1} L^{1-\alpha} = \frac{\alpha Y}{K}$$

$$MPL = (1-\alpha) A K^{\alpha} L^{-\alpha} = \frac{(1-\alpha) Y}{L}$$

Labor productivity and wages

Theory: wages depend on labor productivity. U.S. data:

Time Period	Growth Rate of Labor Productivity	Growth Rate of Real Wages
1960–2019	2.1%	1.9%
1960–1973	3.1	3.2
1973–1995	1.5	1.2
1995–2010	2.8	2.3
2010–2022	1.1	1.3

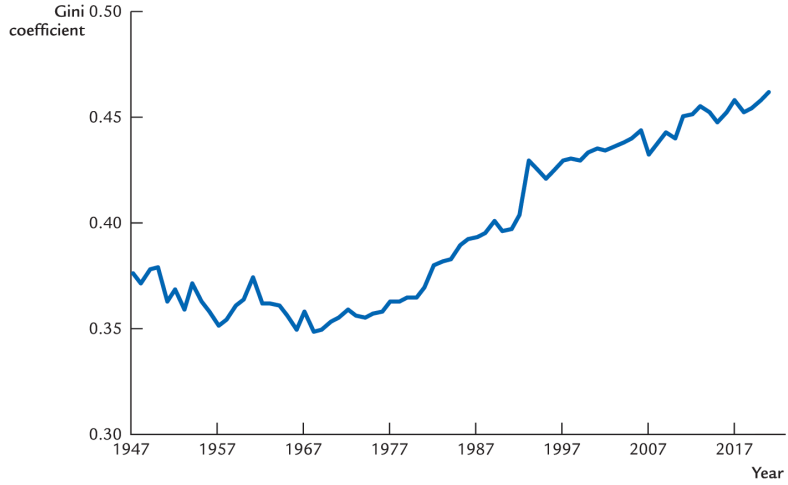
Pitfalls when using productivity and wage data

- According to the neoclassical theory, if productivity and real wages are properly measured, they should track each other closely.
- But the table shows wages have grown at a slower rate than productivity.
- Why?
  - Issues with how we measure wages and productivity.

Income inequality trends

- Since the 1970s, income inequality has increased significantly in the United States and globally.
- One measurement of income inequality is the Gini coefficient.
- A measure between zero (perfect equality) and one (perfect inequality).

Gini coefficient and inequality



Declining labor share and income inequality

- Lower labor share and higher capital share over time has exacerbated income disparity.
- Why?
  - Technological advances have reduced the role of labor and increase the role of capital (higher α in the production function).
  - Market power of firms has increased while worker power has decreased.

Education slowdown and rising inequality

From *The Race Between Education and Technology* by Goldin and Katz:

- Main conclusion of the study is that the increase in inequality is due to educational advancements lagging technological progress.

Skill-biased technological change and inequality

- Technological progress increased the demand for skilled relative to unskilled workers (skills-biased technological change)
- Skills-biased technological change increased skilled workers' wages compared to unskilled ones
- Since the 1970s, technological progress continued, but educational advancements slowed
- Slower growth of skilled workers led to larger disparities in wages between skilled and unskilled workers, increasing inequality

Globalization and inequality

- Trade dynamics and growth in international trade in the United States have increased the demand for skilled relative to unskilled labor.
- These changes have increased the skilled workers' wages compared to unskilled workers, exacerbating income inequality.
- International trade can exacerbate inequality, but most economists believe it benefits economies.

Marriage and inequality

From *Earnings Inequality and the Changing Association Between Spouses' Earnings* by Christine R. Schwartz:

- People with above-average incomes marry people with above-average incomes.
- This *assortative mating* increases inequality.
- Not always the case as this correlation was negative in the 1960s

Changing marriage rates and inequality

- Marriage rates have fallen at a faster rate among individuals with lower educational attainment.
- College-educated workers have higher marriage and lower divorce rates compared to workers without college degrees.

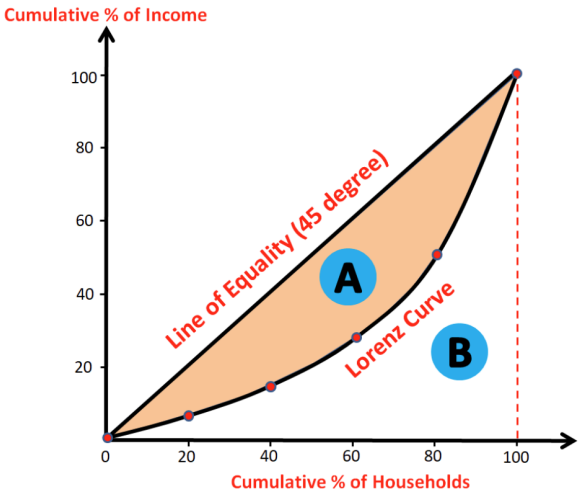
Chapter Summary, Part 1

- Total output is determined by:
  - the economy's quantities of capital and labor
  - the level of technology
- Competitive firms hire each factor until its marginal product equals its price.
- If the production function has constant returns to scale, then labor income plus capital income equals total income (output).

Chapter Summary, Part 2

- Increasing inequality among workers can be explained by numerous factors.
- Declining labor share, education slowdown among skilled workers, the rise of globalization and various cultural changes.

Appendix: Lorenz curve



The **Lorenz curve** plots the cumulative share of **total income** against the cumulative share of the **population**, sorted from poorest to richest.

Reading the graph:

- The 45° line is *perfect equality* — every 1% of people receives 1% of income.
- The more the Lorenz curve sags below the diagonal, the more unequal the distribution.
- Point $(0.6, 0.3)$ reads as: the bottom 60% of the population earns 30% of total income.

back

Appendix: Gini coefficient

The **Gini coefficient** is defined from the Lorenz curve as

$$G = \frac{A}{A + B},$$

where A is the area between the 45° line and the Lorenz curve, and B is the area below the Lorenz curve.

Interpretation:

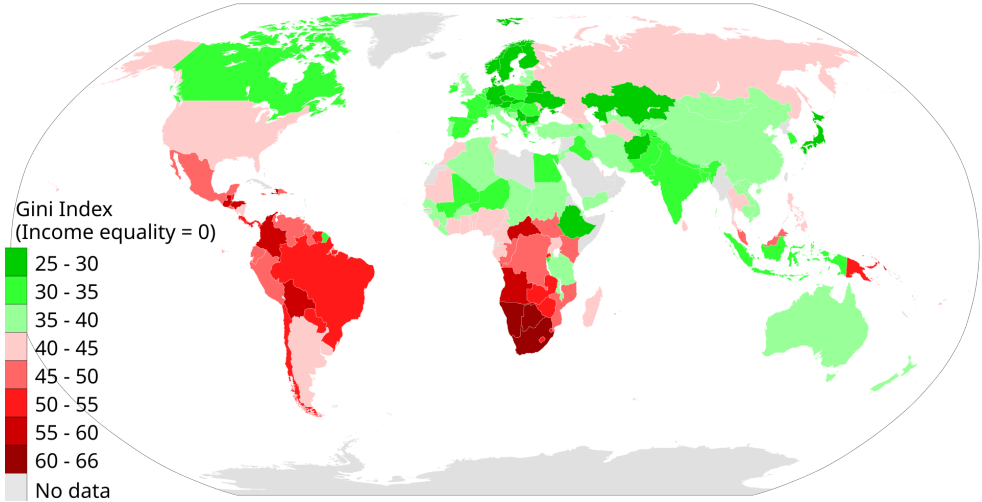
- $G = 0$ — perfect equality (Lorenz curve on the diagonal, $A = 0$).
- $G = 1$ — perfect inequality (one person has all the income).
- Higher G = income more concentrated at the top.

Rough real-world benchmarks (recent):

Country / group	Gini
Nordic countries	≈ 0.25–0.28
Germany, France	≈ 0.30–0.32
United States	≈ 0.40–0.42
China	≈ 0.38–0.47
Brazil, South Africa	≈ 0.50–0.60

The Gini collapses the whole Lorenz curve into a single number — convenient for cross-country comparisons, but it hides *where* inequality lives.

Appendix: Gini around the world



Green = more equal (Nordic, C. Europe); **red / dark red** = more unequal (Latin America, Southern Africa). U.S. sits in the pink 40–45 band — higher than most rich democracies.

Source: World Bank, 2014 (or latest available); *Wikimedia, CC BY-SA.*